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USING NUMBER LINE PROCEDURES AND PEER TUTORING TO IMPROVE THE MATHEMATICS COMPUTATION OF LOW-PERFORMING FIRST GRADERS

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Recommendations for mathematics instruction frequently include the use of manipulatives as a critical component. There are few experimental analyses of teaching strategies involving the use of manipulatives (e.g., the number line). This investigation used a multiple baseline design across three groups of students to examine the effectiveness of an experimental procedure for improving low-performing children's skills in solving missing addend arithmetic problems using the number line. To address concerns about inadequate time for instruction and insufficient practice in most mathematics instruction, trained peer tutors implemented the procedure. The results suggest that student performance improved when trained tutors taught the students number line procedures and gave them feedback on accuracy. Further, social validation data indicate that the students, their tutors, and their classroom teachers liked the procedures.

DESCRIPTORS: number line, peer tutoring, feedback, academic skills, low-performing students

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Arithmetic instruction is a core component of the elementary school curriculum. However, "only 16% of eighth grade students . . . can solve simple equations. The vast majority of them cannot do these kinds of tasks successfully at least 50% of the time" (Anrig & LaPointe, 1989, p. 7). Only 8% of eighth graders can answer mathematical questions that require problem-solving skills (National Assessment of Educational Progress, 1992).

Investigators have identified effective instructional strategies and curriculum design features for improving student performance in mathematics (Carnine, 1994; Carnine, Jones, & Dixon, 1994; Jitendra, Carnine, & Silbert, 1996; Porter, 1989). Darch and his colleagues compared an explicit problem-solving strategy for translating story problems into mathematical equation forms to problem-solving methods presented in four basal mathematics texts (Darch, Carnine, & Gersten, 1984). They found significant positive effects with the explicit strategy but no effects with the methods presented in the textbooks. These findings are consistent with those of Jitendra et al. (1996). To improve mathematics performance, researchers have also advocated presenting skills and concepts sequentially and cumulatively to increase accuracy and retention (Haupt, VanKirk, & Terraciano, 1975; Resnick, Wang, & Kaplan, 1973; Smith & Lovitt, 1975). Other researchers have recommended separating similar mathematical symbols to reduce interference effects (Carnine, 1989; Lovitt &

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Curtis, 1968), emphasizing mathematical relationships to make new learning more meaningful (Carnine et al., 1994; Resnick, 1989), and providing adequate time for practice (Fisher et al., 1978; Porter, 1989).

In 1989, the National Council of Teachers of Mathematics (NCTM) released the Curriculum and Evaluation Standards. The standards' intent was (a) to provide a model for all children to use in discerning the pattern of a mathematics problem, (b) to provide children with the skills to solve the problem correctly, and (c) to assist educators with instructional planning in mathematics. Among NCTM's recommendations is "the use of manipulatives" as a "critical component" in mathematics lessons (Karp, 1990, p. 1). Manipulatives, "concrete models for students to use to solve [mathematical] problems" (Burns, 1996, p. 51), include geoboards, bean sticks, spinners, unifix cubes, Cuisenaire rods, number lines, and other forms of models. The NCTM standards, however, provide neither direction on when and how to use manipulatives nor research-based information on the relative worth of a particular manipulative for instruction. Yet, "if manipulative activities are to be efficient, teachers need guidance on when and how to use" them (Carnine et al., 1994, p. 411).

Perhaps the reason the NCTM standards included limited information on how to use manipulatives is that few experimental analyses on their use exist. In a meta-analysis of 60 studies, Sowell (1989) reported remaining unable to "answer any questions about the nature of situations in which manipulatives might be appropriate nor which manipulatives were most appropriate when" (p. 505).

Some researchers have investigated the effectiveness of the number line as a manipulative. Using Piagetian tasks, Kingma and Zumbo (1988) investigated the relationship between implicit ordinal number comprehension (i.e., seriation and number-numeral correspondence) and explicit ordinal number knowledge measured with number line comprehension tasks. In multiple regression analysis on Piagetian tasks, the combination seriation and number-numeral correspondence showed greater predictive value of number line comprehension task performance at 1 year compared with the predictive value of seriation tasks alone. Others have proposed theoretical analyses for using the number line as an effective manipulative for teaching the ordinal aspects of number (Ernest, 1985; Venger & Gorbov, 1993).

As educators face the challenge of improving student achievement in mathematics and implementing reforms such as those suggested by NCTM (1989), demands on their already limited instructional time may also increase. In contrast, peer tutoring increases the amount of academic learning time available (Greenwood, 1991; Greenwood, Delquadri, & Hall, 1989). "Historically it has been a practical [and inexpensive] means of providing extra help to particular students [and] a means of individualizing instruction" (Greenwood, 1991, p. 111). Investigators have also reported that peer tutoring is effective in increasing levels of student engagement (Greenwood, Carta, & Hall, 1988; Greenwood, Carta, & Kamps, 1990) and improving the mathematics performance of students with differing ability levels (Bentz & Fuchs, 1996; Fantuzzo, King, & Heller, 1992; Fuchs, Fuchs, Bentz, Phillips, & Hamlett, 1994; Greenwood, 1991; Greenwood & Delquadri, 1995; Greenwood et al., 1989).

The current investigation had two purposes. First, we addressed the need for research on the use of manipulatives advocated for mathematics instruction and specific procedures for using them. We analyzed the effects of teaching students to use number lines to solve missing addend problems (e.g., $\_ + 5 = 8$ or $5 + \_ = 8$). Educators have consistently reported missing addend problems to be difficult for many elementary students (Gutstein & Romberg, 1995). We chose the number line because of the lack of previous studies documenting its effectiveness as a manipulative. Second, to address concerns about inadequate time for instruction and insufficient practice in most mathematics instruction, we developed procedures for training peer tutors to teach their classmates to use the number line.

METHOD

Participants

Six girls and 9 boys selected from a class of first-grade students served as participants. They were selected on the basis of their performance on an arithmetic pretest. None of the children had been identified with special needs or referred for special education services. Of these 15 children, the 12 who missed all or all but one of the missing addend problems on the pretest were designated as the "students." They were assigned randomly to four groups of 3 children each. Three boys in the class missed no more than one problem on the entire pretest and no more than one of the missing addend problems. They were designated as "tutors." Each tutor was assigned at random to one of the first three groups: Groups 1, 2, and 3. No tutor was assigned to Group 4.

Setting and Materials

Setting. The first-grade students were taught by two certified elementary teachers and two part-time paraprofessionals in a Chapter I elementary school. At the teachers' request, two first-grade classrooms had been combined into one large class of 44 students to enable the teachers to teach all the first-grade students in the school together. Math instruction occurred for 1 hr on Mondays, Wednesdays, and Fridays and for 30 min on Tuesdays and Thursdays and consisted of daily lessons from the math textbook. In the classroom, all the children sat in small groups of 4, 6, or 8. Although the teachers circulated as the children worked, answering questions when they raised their hands, the teachers did not systematically provide feedback or praise to their students.

During math instruction in the classroom, the students could use the number lines they received during the pretest. The teachers were instructed not to provide any instruction on the use of the number line. Content of the lessons in the classroom varied and reflected the spiral curriculum approach. In this approach, a given mathematical skill is introduced and is followed by another mathematical skill, not necessarily in a logical sequence. As subsequent skills are introduced in the curriculum, they are presented without providing sufficient practice for mastery. The rationale for this approach is that initial mastery is unnecessary because each of the mathematical skills presented to the children will be presented again in the future.

In the hallway outside the classroom, three student desks and a portable chalkboard were used occasionally for small group sessions. The present study was conducted at these three desks during the daily math periods.

Materials. In addition to the math instruction the students received from their classroom teachers, during the study the 12 students also worked on a packet of math problems the experimenters had chosen from the classroom text and other first-grade basal math curricula. The packets for each session consisted of five stapled pages of single-digit addition, subtraction, and missing addend problems presented in horizontal equations. At the top of each page, the date and the child's name were written in pencil. Each page in the packet contained 10 to 20 problems, with none of the problems containing numbers greater than 10. Each child also received a plastic-covered cardboard number line 2.5 cm by 30.5 cm (1 in. by 12 in.) with the numerals from 0 to 10 along a horizontal line. The line was drawn with an arrow at each end. A minus sign was drawn over the arrow pointing left, and a plus sign was drawn over the arrow pointing right.

Measures of Integrity of the Independent Variable

Two measures were used to address procedural integrity. The first measure, following number line procedures, assessed the tutors' and students' compliance with the number line procedures. The second measure, using the number line, assessed the students' use of the number line during their math computation.

Following number line procedures. At the beginning of several sessions in every condition, following number line procedures was assessed for all 15 participants (the 3 tutors and the 12 students). The experimenter asked the student, "Show and tell me how you work this problem on the number line." To be scored as correct, the child's response had to include both the verbal statements and the corresponding number line movements of the 14-step following number line procedures. Omissions or errors in the step sequence were scored as incorrect. None of the children were informed of the results of the scoring. No training, instructions, praise, or corrections were used. All responses were scored using a 14-step sequence checklist. In the example below, the child is following the number line procedures to solve the missing addend problem of 3 + _ = 6:

1. What is the sign? (The child names the operational sign.)
2. Which way do you go? (The child points in the direction of the arrow under the plus sign.)
3. Is the blank before or after the equal sign? (The child says, "Before.")
4. The blank before the equal sign means it's a "tricky one." What do we call this problem? (The child says, "It's a tricky one.")
5. What is the first number? (The child says, "3.")
6. The first number tells you where to start. Where do you start? (The child says, "3.")
7. Put your pencil on the first number. (The child puts his pencil on the 3.)
8. What is the second number? (The child says, "6.")
9. This is what you're looking for. What are you looking for? (The child says, "6.")
10. Let's find 6. Count out loud. (The child moves his pencil along each numeral on the number line between 3 and 6, counting the number of jumps along the number line in sequence.)
11. What number did you find? (The child says, "6.")
12. How many jumps did it take to find 6? (The child says, "Three jumps.")
13. Put that number in the blank. (The child writes the numeral 3 in the blank.)
14. Now read the whole problem. (The child reads, "3 + 3 = 6.")

In this example, the child should have started with a pencil on 3 and jumped the pencil from 3 to 4, touching each numeral with the pencil, drawing an arc between them on the number line. Each subsequent jump, between 4 and 5 and between 5 and 6, would be identical until the child finished with a total of three arcs written on the number line.

Each student and tutor were observed individually and tape recorded while they worked a sample missing addend problem. When the child solved one sample missing addend problem and explained the solution to the experimenter, he or she joined the group in the hallway. After all children in the group had been taped individually, the group observation began. Students in Groups 1 and 2 were observed and audiotaped on 11 sessions, Group 3 students on nine sessions, and Group 4 students on seven sessions. The tutors for Groups 1, 2, and 3 were observed and audiotaped on 15, 14, and 10 sessions, respectively.

Using the 14-step checklist, independent reliability checks for each of the sessions yielded 100% agreement on children's scores ranging from 14% (i.e., two steps correct) to 100% of the 14-step procedures correct. Following the number line procedures for each of the children in the four groups and the tutors yielded the following mean scores: Group 1, 86%; Group 2, 91%; Group 3, 84%; Group 4, 17%; and the tutors, 98%. Independent reliability checks of the experimenter's adherence to the 14-step training sequence were made four times from the audiotapes of the training sessions and resulted in 100% agreement.

Using the number line. Observations of the children using the number line were recorded on a form divided into 10-s intervals. To be scored as using the number line, the child had to be observed to move a finger or pencil along the number line while computing a problem. The child was not scored as using the number line for merely picking up the number line, touching it, or holding it in one or both hands.

Each group of 3 students was observed for 9 min every day, except on 3 days when no data were taken because 2 students in the group were absent. Each individual student in the group was observed for a total of 3 min. The first student in Group 1 was observed for the 1st minute, the next student for the 2nd minute, and the 3rd student for the 3rd minute. This observation sequence was repeated three times for Group 1. The same observation procedure was followed with the other three groups.

Using the number line was recorded in each 10-s interval in which it occurred using partial-interval scoring. If at any time in the interval the child was observed using the number line according to the definition, the observer scored an occurrence. The percentage of intervals of occurrence was calculated by dividing the total number of occurrences by the 18 intervals possible during the 3-min observation of each student.

Reliability checks were made on 12 sessions and yielded a range of agreement within groups between 58% and 100%. The 58% agreement occurred the first day of the reliability checks; following the observation, the observers reviewed the definitions. Subsequent agreement between observers ranged from 78% to 100%. Group means for total reliability were 82%, 86%, 84%, and 85% for Groups 1, 2, 3, and 4, respectively.

Observers and observer training. The primary observer for all the data collected during the study was the first author. This observer trained the two reliability observers.

Dependent Variables

The dependent variables obtained for all class members, the 12 students, the 3 tutors, and the classroom teachers were daily problems correct, problems stamped, and the pretest and posttest.

Daily problems correct. During the 9 min of the daily experimental session, the students worked on solving the equations in their packet. All students worked through the problems in their packets in the same order. Because they worked at different rates, however, students within a group could be working on different pages in the packet during a given session.

Following the 9 min of observation for all 12 students, the number of problems correctly computed was counted and divided into the two categories of missing addend and nonmissing addend problems (i.e., all other problems). The number of missing addend problems worked in a given session ranged from 0 to 25 across students (range, 0% to 80%). Number of missing addend problems correct was calculated by dividing the number correct by the total number of missing addend problems worked during a session. Agreement on independent reliability checks for all sessions for missing addend problems correct ranged from 93% to 100% (M = 96%). Number of nonmissing addend problems correct was calculated by adding together all other correctly computed problems and dividing them by the total number of nonmissing addend problems worked during a session. Agreement on independent reliability checks for all sessions for nonmissing addend problems correct ranged from 96% to 100% (M = 98%).

Problems stamped. Each tutor was instructed to stamp all correctly computed problems on the students' papers with a rubber ink stamp, without saying anything about accuracy. The tutors could stamp either missing addend or nonmissing addend problems correct. The rubber stamp procedure was designed to prompt the tutors to contact each of their tutees and to provide a permanent product measure of feedback. The total number of problems stamped by the tutor was scored each day for those students with tutors, following the 9-min observation. The independent reliability checks for correct problems stamped ranged in agreement from 87% to 100% (M = 96%).

Pretest and posttest. All children in the class took the pretests and posttests. The identical 44-item tests contained no numerals greater than 10, with 9 addition, 11 subtraction, and 12 missing addend problems distributed randomly throughout. Each child was also given a number line to use. Percentage correct was calculated for each child for each problem category on the pretest and the posttest.

Consumer Satisfaction Ratings

To determine how well they liked the procedures, all 15 of the students and tutors were asked to respond to three questions. Using a 3-point Likert-type response scale, the first question asked them to indicate how difficult or how easy using the number line made the "tricky ones," (the missing addend problems) with 1 = *harder than before*, 2 = *about the same as before*, or 3 = *easier than before*. The second question used a yes-no response format and asked them whether they liked the number line. Using a 3-point Likert-type scale of *a lot*, *somewhat*, or *not at all*, the third question asked the children how much they liked "coming out in the hall and working with some of your friends."

A written questionnaire was used to assess the teachers' reactions to the students' and tutors' behavior in the classroom, use of the number line, and math assignment completion. The teachers completed the questionnaire twice, once at the beginning of the study and once at the end. Sample questions included the following: "Do the peer tutors complete their classroom math assignments?" "Do the children that are working with the tutors complete their math assignments?" "Do the children working with the tutors use their number lines in the classroom?" and "Are the students cooperative during the school day?"

Experimental Design and Procedure

The experimental design consisted of a combination of a multiple baseline (Risley & Wolf, 1973) across the three groups, with tutors plus a no-tutoring group, and with a pretest and posttest comparison within each group (Campbell & Stanley, 1963). The sequence of experimental conditions for the tutored groups was as follows: pretest, tutoring, tutoring with training (only for

Group 1), tutoring with training and feedback, and posttest. The sequence of conditions was arranged to analyze the effects of untrained tutoring, tutors trained in specific number line procedures, and trained tutors who also provided feedback on accuracy. The untutored group was used to analyze the effects of simply providing students with a number line and extra practice but no training, a condition most similar to classroom instruction. The pretest and posttest given to the class provided information on the computational skills of the 21 remaining students who did not participate in the study and received only classroom instruction in math.

Pretest. The first author administered the 44-item pretest to all the students in the class during a regular math period. Each child received a number line and was instructed to try his or her best without any help from the teachers.

Tutoring (T). At the beginning of the classroom math period, the first tutor and the 3 students in his group went out to the desks in the hallway. On each desk there was a number line set out for them to use and at least five pages of math problems. When the 3 children were seated, the experimenter told the tutor, "I want you to help your friends with their math but don't give them any answers." The experimenter made the announcement loudly enough so that all the students could hear. Next the tutor told the students that he would help them without telling them the answers, and the tutees subsequently repeated the tutor's announcement. The experimenter gave no instructions to either the tutor or the 3 students about the number lines.

After 9 min of observation, the experimenter told the students to stop working and noted their stopping point on the page. Then the 3 students and their tutor returned to the classroom to work on their regular classroom math assignments with the teacher. The experimenter provided no instructions, suggestions, or feedback to the teacher at any time during the study. Following Group 1, Group 2 and then Group 3 went out into the hall at their assigned times. All groups followed the same procedure. The tutoring condition was applied to Group 1 for four sessions, to Group 2 for 13 sessions, and to Group 3 for 20 sessions.

Tutoring with training (TT). The 3 tutors received training on following number line procedures, and they in turn trained their students. The experimenter trained the tutors individually in an adjoining classroom and completed training in approximately two sessions. First the experimenter modeled the correct step sequence for solving missing addend problems on the number line (i.e., following number line procedures). The tutor was next asked to perform the correct steps and describe the solution to the experimenter. The tutor was then given a different sample missing addend problem and asked to solve it with the number line. Regardless of the tutor's errors on either problem, he worked only two sample problems in each session. During the training, the tutor's last solution of a missing addend problem was tape recorded and scored. The experimenter made no comments about incorrect responses during training, but she did praise the tutor for correct responses. Following the 10-min training session, the tutor went out into the hall with his 3 students.

While the 3 students were seated at their desks in the hall, the experimenter instructed the tutor to explain the solution of two problems on the chalkboard, using a large number line drawn on the board. The tutor explained two new problems daily: one with the missing addend preceding and one with the missing addend following the plus sign. The tutor told the 3 students to look at the board. He went through the number line procedures orally for each problem. Once the tutor began his explanation, the experimenter provided no further instructions or coaching. When the tutor had finished, he instructed the students to begin working the daily assignment. At this time, the tutor was instructed to go to each student and explain the number line procedures individually. The tutor usually divided his time among the 3 students. When 1 of the children failed to receive any instruction, however, the observer asked the tutor to help that child. Just as in the tutoring condition, the tutor was instructed to give assistance, not to furnish answers.

Only the students in Group 1 received this condition. Although the tutor was performing all 14 steps of following the number line procedures correctly after the 2nd day of training, he was failing to contact his tutees. The TT condition was terminated for Group 1 because the students' error levels of both missing addend problems correct and nonmissing addend problems correct had decreased below baseline levels. The feedback procedure in the next condition was added to increase the tutor's contact with his tutees.

Tutoring with training and feedback (TTF). The training on following the number line procedures for the tutors in this condition was identical to the training in TT. While all 3 students were seated at their desks, the tutor explained the solution of the two problems on the chalkboard using a large number line drawn on the board. After the tutor demonstrated the correct procedures for solving the two sample missing addend problems, he was instructed, "Show each of your friends how to work the 'tricky ones' [missing addend problems] with the number line. After you have helped them all, take a rubber stamp and stamp the problems they have right." Tutors were prompted only once at the beginning of each session in this condition. At the end of the observation, the children and the tutor returned to the classroom and worked on class math assignments.

No tutoring. The students in Group 4 in the no-tutoring condition followed a standard procedure every day. The 3 students went into the hallway at their scheduled time, sat down at a desk, and worked math problems. Each student had a number line but received no instructions for its use. The students were told to work the problems as well as they could without help from the teacher. At the end of the 9-min observation, they turned their papers over and returned to the regular classroom math period.

Posttest. During the last week of the study, all 44 first-grade children in the class received the posttest while seated at their desks in the classroom. Each child also received a number line to use. The posttest was identical to the pretest. Again the children were instructed to try their best without any help from the teachers.

RESULTS

Figure 1 provides the accuracy with which the four groups of students computed mathematics problems during each session.

Missing addend problems correct. During the tutoring condition, Group 1 students correctly computed a mean of 28% of the missing addend problems (range, 0% to 66%). The mean values in this condition reflect the higher performance of only 1 of the 3 members of the group. The other 2 students maintained mean percentages of accurately computed problems below 5%. Group 2's mean in this condition was 6% (range, 0% to 25%). Group 3's mean was 7% (range, 0% to 23%).

In the tutoring with training condition provided only to Group 1, there was a slight increase from the previous condition's mean to 34% (range, 0% to 75%). One student continued to perform at T condition levels of accuracy throughout the TT condition.

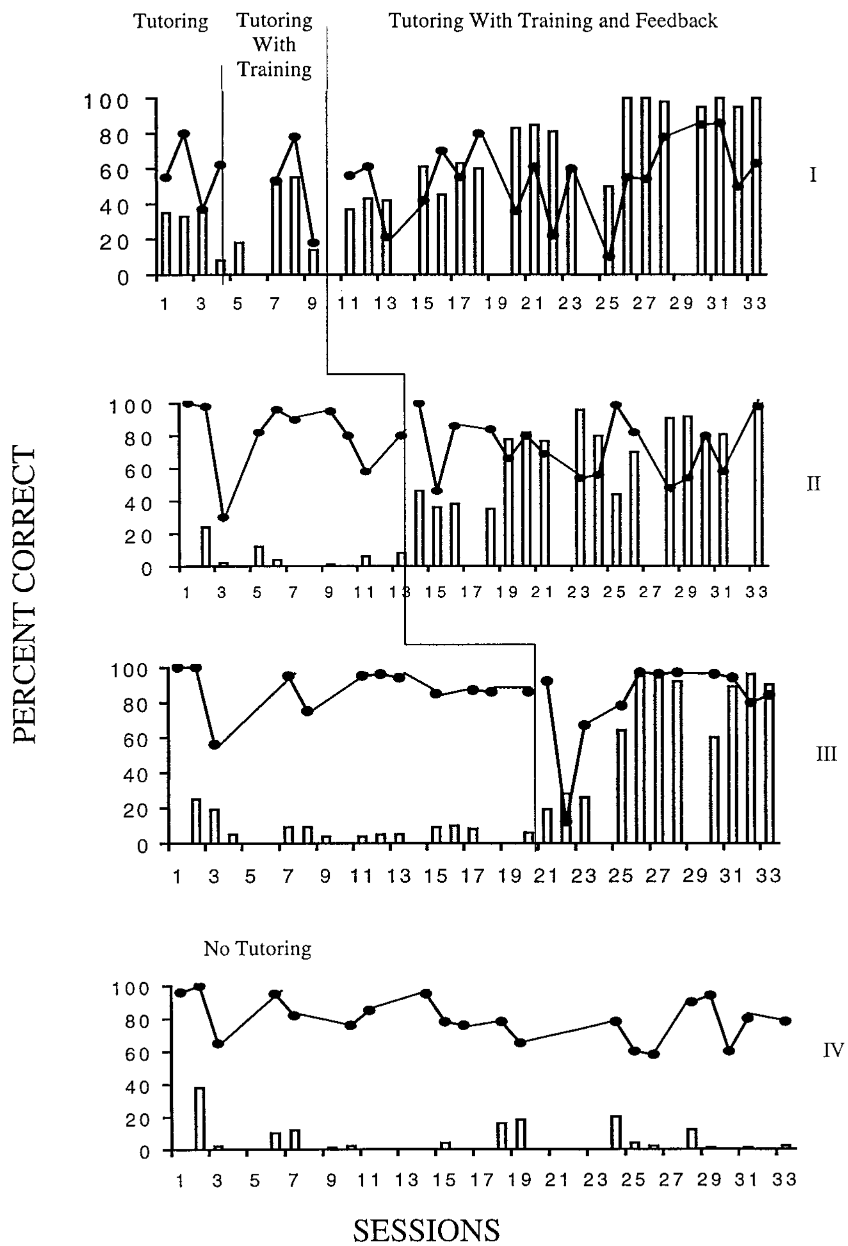

Figure 1. Average daily performance on problems correct for Groups 1 (top panel), 2 (second panel), 3 (third panel), and 4 (control in fourth panel). The bars represent missing addend problems, and the connected points represent all other problem types. Missing data indicate those sessions during which no data were collected because 2 of the students in a group were absent.

In the tutoring with training and feedback condition for Group 1, the group's mean performance improved over previous condition levels on Session 15. The group maintained its high performance throughout the remaining sessions. Group 1's mean for TTF was 72% (range, 13% to 100%). Group 2's performance showed immediate improvement above T condition levels during the initial sessions in TTF. Their performance subsequently improved further and was maintained at a high level of accuracy for the rest of TTF, with a mean of 71% (range, 5% to 100%). In Group 3, performance improved slightly in the first three sessions of TTF to a mean of 73% (range, 10% to 100%), then increased to levels substantially above those in the T condition and was maintained. All students in Groups 2 and 3 performed with slight individual differences, but each group's mean data were representative of the individual performances.

Group 4, with classroom instruction and a number line but with no tutor or number line training, had a mean of 7% correct on missing addend problems (range, 0% to 22%). The group's performance on missing addend problems remained comparably low across all sessions.

All other problem types. For all students, accurate computation of all other problems remained stable throughout the study. Group 1 students averaged about half of the other problems correctly computed with a mean of 59% in T, 50% in TT, and 53% in TTF. Group 2 students correctly computed 81% of the other problems in T and 75% in TTF. Group 3 students correctly computed 88% of the other problems in T and 83% in TTF, and Group 4 students averaged 81%.

Posttest. Seven of the 9 tutored students, or 77%, clearly improved with the TTF treatment. One of the 7 increased his pretest performance from 0% to 50%. The mean posttest score for the other 6 was 80%. The posttest scores for all 3 of the untutored students in Group 4 was 0%. The tutors' mean percentage correct on the pretest was 97%, and all 3 tutors correctly computed 100% of the missing addend problems on the posttest. The 21 students who did not participate in the study computed a mean of six missing addend problems correctly (50%) on the pretest and eight (75%) on the posttest.

Social validity. Student interview data were divided into three categories: those from the tutors, the tutored students, and the untutored students. All 3 tutors and 8 of the 9 tutored students responded that the "tricky ones," missing addend problems, were easier to work after training than before training. None of the untutored students responded that the "tricky ones" were "easier to work than before."

Each of the three groups of children with tutors responded identically to the last two interview questions. On a yes-no question about liking the number line, all 3 of the tutors, 8 of the 9 tutored students, and 1 of the 3 nontutored students answered "yes." All 3 tutors, 8 of the 9 tutored students, and 1 of the 3 nontutored students responded "a lot" when asked how much they liked working in the hallway.

The teachers' responses to questions about the tutors indicated that the tutors continued to complete their classroom math assignments and to cooperate with the teaching staff throughout the school day. The classroom teachers noted no unfavorable changes in the tutors' classroom deportment during the entire study.

The teachers reported that the study group completed classroom math assignments "sometimes" before the study and "most of the time" during the study. The teachers also reported that although the 9 tutored students had never used their number lines in the classroom before the study, they did use them "most of the time" during the study.

DISCUSSION

Tutoring with systematic number line procedures and feedback was effective in training first-grade students to compute missing addend problems. Number of missing addend problems correctly computed by the students in Groups 1, 2, and 3 increased when the TTF procedure was used, but no comparable increase was noted for the untutored children in Group 4.

The pretest and posttest data suggest that no effects resulted solely from providing a number line to the students in Group 4 or to the 21 students who did not participate in the study. No effects were observed with the presence of a tutor told to "help his friends" and the number line with Groups 1, 2, and 3. Further, when the tutor for Group 1 was trained on the instructional procedures alone, the initial increase failed to be maintained. Only when the feedback procedure was added to tutor training with the number line were the desired effects obtained.

Although all students in Group 1 increased the number of missing addend problems correctly computed, the variability of the data in this group reflects the performance of 1 of its students. In the T condition, this student (H) immediately increased his percentage of correctly computed missing addend problems over his score of 0% on the pretest. The mean percentage correct across sessions of 28% in this condition was considerably higher than the 5% mean correct across sessions for the other students in Groups 2 and 3 in the T condition. When the TT condition was implemented with Group 1, the mean increased to 34%, again reflecting the increase of a single student. For the other 2 students in this group, percentage correct did not increase until the TTF condition. One of these students (V) improved very gradually in this condition. Although the procedural fidelity data for this student show that he could perform at least 12 of the 14 steps correctly during training, he scored only one missing addend problem correctly on the posttest. It is possible that this student required greater opportunities to practice his newly acquired skill than the other low-performing children in the group.

The posttest score of 1 of the tutored students in Group 2 (D) failed to improve from his pretest score of 0%. This same student was the lone negative voice during the social validity interview, reporting not liking the number line, not liking coming out into the hall, and not finding the number line helpful. Although he improved his computation of missing addend problems during training, it is not known why he failed to maintain his performance on the posttest.

Research on mathematics instruction recommends that instruction (a) build on prior knowledge, (b) focus on critical features of the algorithm, (c) provide explicit teaching demonstrations, (d) present skills sequentially, (e) separate similar mathematical symbols to reduce interference, and (f) provide adequate opportunities for practice (Carnine et al., 1994). The specific number line procedures incorporated the first five recommendations, and peer tutoring provided the sixth. The number line procedures emphasized the students' prior knowledge of the operational signs, numeral recognition, and equations. The critical feature of missing addend problems (i.e., the location of the missing quantity in the equation in relationship to the equal sign) was therefore added to their prior knowledge, using the verbal prompt of "tricky one" to distinguish them. The plus and minus symbols above the directional arrows at each end of the number line provided prompts for cuing which direction to "jump" to solve the equation. The step-by-step number line procedures provided the students with explicit demonstrations of how to use the number line, procedures that were missing from classroom instruction and from the classroom math text. Because the students and teachers liked the procedures and reported that they were helpful, the application of these number line procedures to other classrooms is promising.

Diagrams of number lines have existed in mathematics basal textbooks for over 20 years (e.g., Duncan, Capps, Dolciani, Quast,

& Zweng, 1975; Immerzeel & Wiederanders, 1973). Theoretical rationales also exist for using the number line to teach numeral concepts to young children (e.g., Ernest, 1985; Venger & Gorbov, 1993). In the present study, tutoring with systematic number line procedures and feedback provided experimental validation of the number line's effectiveness as a mathematical manipulative. The results support the teaching value of the number line procedures and extend the empirical base for their recommended use (e.g., Carnine, 1994). An additional advantage is that the procedures were sufficiently simple to be used by peer tutors.

Using peer tutoring to increase students' opportunities for practice and for increasing their academic engagement time in mathematics was highly effective for all of the targeted low-performing students. These findings are consistent with previous research (Delquadri, Greenwood, Whorton, Carta, & Hall, 1986; Greenwood, 1991; Greenwood et al., 1989). The favorable effects of using trained tutors rather than untrained tutors are also supported by previous findings (Bentz & Fuchs, 1966; Fantuzzo et al., 1992; Fuchs et al., 1994).

This study did not separately evaluate the effects of feedback from those of the number line procedures, but anecdotal evidence suggests that feedback served as a prompt for the tutors to implement the procedures with their tutees. Although the tutor for Group 1 quickly acquired the number line procedures, he initially required prompting by the experimenter to actually use these procedures with his tutees. To avoid prompting the tutor to use the procedures throughout the tutoring sessions, the experimenter introduced use of the rubber stamp. The instructions to stamp problems correct required the tutor to actually do something with each of his tutees. Using a specific strategy to increase the effectiveness of the interaction between the tutor and his tutees is consistent with the work of others (Bentz & Fuchs, 1996; Fantuzzo et al., 1992; Fuchs et al., 1994; Greenwood, 1991).

The performance of the students in Group 4 declined across all problem types over the length of the study. This decline in performance over time is consistent with earlier findings that the time students spend making errors correlates negatively with achievement (Fisher et al., 1978). It also provides evidence of the need for specific procedures for mathematics instruction.

Several limitations might be addressed in future studies. First, occasional probes should be used during training to determine each individual student's acquisition of skills and to vary the amount of individual practice as required. In addition, measures of the rate of problem completion and accuracy would permit investigators to monitor individual increases in fluency. Previous findings suggest that certain minimum rates of correct responding appear to be necessary for skill retention and fluency (Binder, 1996). To build a better case for the strength of the number line procedures themselves, it would be useful to collect data on the tutors' fidelity to the training procedures during their modeling of the two sample problems during the actual tutoring sessions. Equally important would be direct observations of classroom math instruction.

Second, although the untutored students served to provide a comparison, the lack of a baseline may be a limitation in the present study. Another limitation may be the lack of a feedback-only condition. Other evidence suggests that feedback alone may be a critical component in effective peer tutoring (Fantuzzo, Davis, & Ginsburg, 1995; Fantuzzo et al., 1992).

Third, in some instances tutors have benefited more from the tutoring than their tutees (e.g., Jenkins & Jenkins, 1981). Using both high- and low-ability students as tutors might illuminate this issue. Training all of the students in a class to use the number line procedures also would provide additional information on the procedures validated in this study.

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STUDY QUESTIONS

1. Define *manipulatives* and give an example not already provided by the authors.

2. How were participants selected as "students" and "tutors"?

3. How did the authors measure treatment integrity?

4. What were the dependent variables?

5. Describe the experimental conditions. What was the purpose of the control group?

6. Summarize the results obtained during the various tutoring conditions.

7. The authors reported that much of the variability in the group data reflected the performance of one individual. Give some reasons that may account for inconsistent performance across individuals.

8. What additional data (not collected during the study) may have been helpful in evaluating the effects of the program?

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